Buying Voters with Uncertain Instrumental Preferences

Charles Louis-Sidois and Leon Musolff*

June 10, 2021

Abstract

This paper analyzes a vote-buying model where voters with uncertain preferences over outcomes vote on a proposal important to the buyer. We characterize the cheapest combination of bribes that guarantees the proposal's passing in different voting environments. When voting is simultaneous, we find that the number of bribes increases with the uncertainty in members' preferences. For both simultaneous and sequential votes, the vote buyer bribes a substantial supermajority of members. Each member accepts because he anticipates that the proposal will pass regardless of his vote. We discuss the committee design that maximizes the cost of capture. In small committees, sequential voting increases cost, while the opposite is true for large committees.

JEL Classification: D71, D72

*Louis-Sidois: WU (Vienna University of Economics and Business), Welthandelsplatz 1, Vienna 1020, Austria (email: charles.louis-sidois@wu.ac.at). Musolff: Department of Economics, Princeton University, Julis Romo Rabinowitz Building, Princeton NJ 08544 (e-mail: lmusolff@princeton.edu).

We thank Alessandro Riboni, Ana Luiza Dutra, Andrea Mattozzi, Christoph Rothe, Elia Sartori, Emeric Henry, Ernesto Dal Bo, Evgenii Safonov, Faruk Gul, Françoise Forges Franz Ostrizek, Hans Peter Grüner, Ian Ball, Jeanne Hagenbach, Joao Thereze, Jonas Müller-Gastell, Leaat Yariv, Matias Iaryczower, Margaret Meyer, Nicole Kliewer, Pietro Ortoleva, Rachel Kranton, Roland Benabou, Sergei Guriev, Stephen Morris, Thomas Romer, Thomas Troeger, Wolfgang Pesendorfer, as well

1 Introduction

Political leaders often want to pass laws going against legislators' interests. For example, consider a government introducing a bill limiting dual mandates for members of a parliament.[1] If the bill passes, legislators are worse off because they are forced to quit their other offices. To circumvent legislators' opposition, the government can reward those who vote in favor of the bill, for instance with positions in legislative committees or with investments in legislators' districts.

We propose a vote-buying model to study such situations. A proposal favoring the interests of a vote buyer is submitted to a vote. Voters value the outcome and prefer the proposal not to pass. The vote buyer makes payments in exchange for voters' support. Offers are simultaneous, and payments can only depend on individual votes. Our key innovation is to introduce uncertainty in voters' preferences in this setup. In the above example, each legislator is the best informed of his future (re)election prospects for other offices. Thus, a legislator's valuation is uncertain for the government and other legislators. This paper shows that the vote buyer can exploit this uncertainty: some distributions of bribes induce an inevitable coordination failure of committee members, implying that the proposal is always accepted. Our mechanism relies on pivotal probabilities and is illustrated by the following example.

Motivating Example

Consider a committee of three members voting simultaneously on a proposal with a simple majority rule. Committee members dislike the proposal and get a disutility $v_i \sim U[0,1]$ if the proposal is accepted. Payoffs are drawn independently and privately.

A vote buyer (feminine pronoun) is interested in making the proposal pass. She publicly commits to paying a bribe b to some members if they individually vote in favor. She knows the distribution of members' disutilities but does not observe their

as discussants at seminars at Princeton, Science Po, Mannheim and Yale for many insightful comments.

[1]A dual mandate, or double jobbing, is the practice in which elected officials serve in more than one elected or other public position simultaneously. Such a law was passed in France in 2013 when more than 80% of parliament members held another office.

realized types. Moreover, assume that the proposal is critical to the vote buyer so that she wants to guarantee that it will pass. Subject to this condition, she aims to minimize the cost spent on bribes.

We compare two possible strategies for the vote buyer. First, suppose she tries to bribe the minimal winning coalition, i.e., she offers a bribe to two members. We assume the unbribed member plays his weakly dominant strategy and votes against the proposal. Then each bribed member is sure to be pivotal if the proposal passes. Thus, as long as $b < 1$, there exists an equilibrium where members vote against the proposal if v_i is large enough. So, to guarantee certain approval, she should offer a bribe of 1. Doing so yields a total cost of 2.

Now, suppose the vote buyer proposes a bribe b to all three members. We will show that as long as $b > \frac{8}{27}$, there is no equilibrium where members vote against the proposal with positive probability; in other words, we will show that buying a third member is cheaper for the vote buyer. To do so, note that, for each member, voting in favor of the proposal guarantees payment of the bribe. However, if the member is pivotal (i.e., if exactly one other member votes for the proposal), it also leads to the proposal's adoption. Denoting the pivotal probability by π, member i votes for the proposal if $b > v_i \times \pi$.

The equilibrium of the voting stage takes a cutoff form: members vote against the proposal if their disutility is larger than a threshold. For the time being, focus on symmetric strategies and call the common cutoff $\overline{v}$. Then $\pi(\overline{v}) = 2\overline{v}(1 - \overline{v})$ and an interior equilibrium satisfies

$$b = \overline{v}\pi(\overline{v}).$$

In Figure 1, we plot the right hand side of this equation as a function of $\overline{v}$. We see that for small bribes like b^1, two equilibria exist with interior cutoffs $\overline{v}_1$ and $\overline{v}_2$. Moreover, there is a third equilibrium where all members accept the bribe: committee members are not pivotal and have no incentive to deviate. Throughout the paper, we assume that the vote buyer expects bribed committee members to play the equilibrium in which the proposal is rejected with the highest probability: e.g., faced with b^1, they would play $\overline{v}_1$ as this lower cutoff implies a lower probability of passing the proposal.

When b is larger than the maximum of $v\pi(v)$, the third equilibrium – where the proposal is necessarily accepted – is the only equilibrium of the voting subgame. Here,

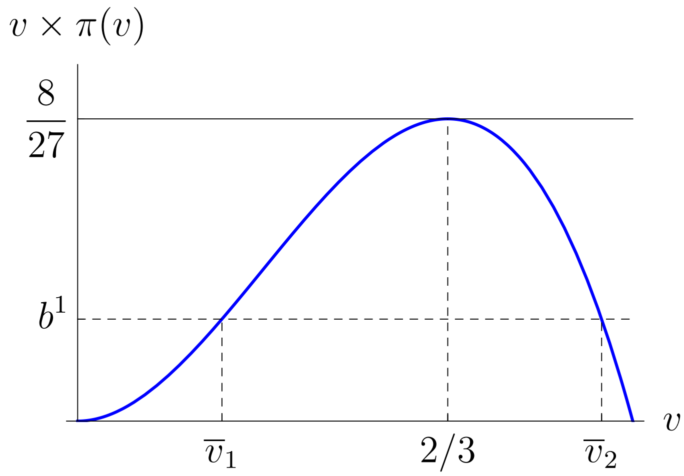

Figure 1: The Structure of Equilibrium in the Voting Subgame

Notes: As the cutoff used by other members changes, so does the function $v\pi(v)$ (solid blue). The function is maximized for $v = 2/3$ and the value of the maximum is $8/27$. For a given bribe b^1 below $\frac{8}{27}$, there are three equilibria: two interior and one in which all members vote for the proposal. For bribes above $\frac{8}{27}$ only the latter survives.

the maximum is $\frac{8}{27}$ and is reached for $v = 2/3$. Thus, it is sufficient to pay slightly more than $\frac{8}{9}$, which is the total cost of bribing all three members, to guarantee there is no equilibrium where members vote against the proposal. Intuitively, bribing a supermajority reduces members' pivotal probability, which induces them to accept smaller bribes in exchange for their votes.

This paper characterizes the cheapest combination of bribes guaranteeing certain passing of the proposal for general distributions of disutility, committee size, and majority requirement. Section 2.1 considers simultaneous voting and focuses on members playing symmetric strategies and bribes of equal values. A key component of our analysis is the dispersion of the distribution of disutilities, which reflects the uncertainty in members' preferences. We establish that the equilibrium number of bribes increases with dispersion. Moreover, the vote buyer always bribes a substantial supermajority: even with minimal dispersion, the coalition bribed is 50% larger than minimal winning. For distributions sufficiently dispersed, all members are bribed.

We allow members to play asymmetric strategies in Section 2.2. Depending on the distribution, there may exist asymmetric equilibria where the proposal can be rejected when members receive the bribes derived in Section 2.1. In particular, if the distribution of types has little dispersion, members can coordinate on an asymmetric equilibrium where the proposal is rejected if disutilities are sufficiently large. However, such coordination is impossible when types are too uncertain, and the proposal is necessarily accepted. Section 2.3 considers bribes of different values. A large dispersion favors a deviation of the vote buyer, and we show an example where a "divide and conquer" strategy based on small and large bribes lowers the cost. However, in the above example with $v_i \sim U[0,1]$, bribes of equal values are indeed the cheapest solution to guarantee that the proposal passes.

We study members voting sequentially in Section 3 and find that the vote buyer can similarly exploit pivotal considerations. The cheapest bribing scheme guaranteeing that the proposal passes again consists of offering the same bribe to all members when $v_i \sim U[0,1]$.

The effect of committee design on the cost of capture is discussed in Section 4. For simultaneous as well as sequential voting, increasing the majority requirement for a given number of members increases the capture cost because members can be pivotal with a larger probability. Moreover, a proportional increase of majority requirement and number of members results in a less than proportional increase in capture cost because members are less likely to be pivotal in a large committee. Finally, compared to simultaneous voting, sequential voting yields an increased cost of capture if the committee is small, while the opposite is true for large committees.

Our setup primarily applies to decision-making in organizations. Suppose a CEO wants to persuade board members to make a decision favoring his interests – such as a wage increase. The CEO can obtain members' support in exchange for small favors if they expect the wage increase to be approved regardless of their vote. The same mechanism can also explain why ruling parties are reelected with large majorities in non-democratic countries: buying the support of a small majority would be expensive because each individual (or faction) can be decisive, but small rewards are sufficient if a big victory is expected. Finally, our model has implications for vote-buying in juries and committees of experts (like FDA committees), and lobbying in a legislature.

We contribute to the literature on vote-buying by combining a single vote buyer with voters who care about the vote's outcome but are uncertain about each other's valuation. This combination is novel, though there is literature on each of the ingredients.

Several papers study vote-buying when voters have publicly known preferences over outcomes. In such a setup, Dal Bo (2007) shows that a vote buyer can bribe a committee at no cost by conditioning the bribes on the full voting profile. She proposes to pay an infinitesimal amount if voters are not pivotal and a large bribe if votes are decisive. Similarly, the models of Rasmusen and Ramseyer (1994) and Dahm and Glazer (2015) feature some equilibria in which arbitrarily small bribes are accepted by a supermajority because no voter is pivotal. Cheap capture also occurs in Genicot and Ray (2006) and Chen and Zápal (2020) where the vote buyer approaches voters sequentially and exploits the timing of offers to induce a coordination failure. By contrast, we allow voters to coordinate on their preferred equilibrium. Furthermore, we exclude contracts based on the joint realization of votes and focus on simultaneous offers.

We show that uncertainty in voters' preferences is sufficient to generate a coordination failure. This failure relies on pivotal probabilities. Levine and Palfrey (2007) test the model of Palfrey and Rosenthal (1985) and provide experimental evidence for the impact of the probability of being pivotal on voter turnout and Shayo and Harel (2012) show that pivotality affects voting choices.

We focus on the probability of a vote being decisive and do not consider information aggregation (Feddersen and Pesendorfer 1996, 1997, 1998). Interestingly, Feddersen and Pesendorfer (1998) highlight that unanimity, which in our setup maximizes capture cost, makes information harder to aggregate. Henry (2008) and Felgenhauer and Grüner (2008) combine vote-buying and information aggregation. In Henry (2008), each committee member receives a signal about the quality of a common value proposal. Bribes determine the number of voters who vote informatively, shaping voters' inferences conditional on being pivotal. Similarly, in Ekmekci and Lauermann (2019) an election organizer chooses turnout to manipulate the information aggregated in equilibrium.

As it relies on pivotality, the induced coordination breakdown in our model is not present in the literature on vote-buying with expressive voters: e.g., in Zápal (2017), voters' responses to a bribe are uncertain, but pivotal considerations are excluded because they do not take into account the effect of their vote on the outcome. Similarly, following Groseclose and Snyder (1996), a strand of the literature assumes expressive preferences and introduces a second vote buyer. These papers include Banks (2000), Dekel et al. (2008), Morgan and Várdy (2011) and Iaryczower and Oliveros (2017). They find that the first mover bribes a large coalition to increase the cost for the follower.

Our paper proposes a new explanation for supermajorities. While early theories of coalition formation predicted minimal winning coalitions (Axelrod, 1970), some later papers predict supermajorities (Koehler, 1975; Weingast, 1979; Shepsle and Weingast, 1981; Baron and Diermeier, 2001). The closest to us is Carrubba and Volden (2000), in which a larger-than-necessary coalition ensures no member can prevent the costly (to him) passage of other members' bills. Supermajorities are also found in the literature on legislative bargaining (Volden and Wiseman, 2007; Tsai and Yang, 2010; Dahm et al., 2014); for an overview, see Eraslan and Evdokimov (2019). For instance, Norman (2002) characterizes the non-symmetric equilibria of the classical model of Baron and Ferejohn (1989) and shows some proposals can be approved by all.

Moreover, Chen and Eraslan (2013, 2014) look at the other side of the problem and study a vote-selling model where voters with uncertain preferences send messages to the vote buyer to influence the proposal. Finally, we are also related to the larger literature on group incentives. In Winter (2004) and Winter (2006), agents separately perform individual tasks for a larger project, which succeeds if all agents succeed. In case of success, the principal rewards agents who support the project. Contributions are simultaneous in Winter (2004) and sequential in Winter (2006). Our paper differs as players' preferences are uncertain. Winter's papers consist in preventing asymmetric equilibria where the project fails and establishes that discriminatory rewards can be optimal. With uncertainty, asymmetric equilibria cannot be sustained, and equal rewards are preferred.

2 Simultaneous Voting

We consider a committee of n members voting simultaneously on a proposal. The proposal is accepted if at least m members vote for it. Committee member i privately draws the disutility $v_i \overset{iid}{\sim} F(\cdot)$ he obtains if the proposal is accepted. Before the voting stage, a vote buyer in favor of the proposal publicly chooses a bribing scheme $(b_1, ..., b_n)$, where b_i is the payment for member i if he votes in favor. We assume that the vote buyer's valuation for the proposal is large, so that her objective is to minimize the amount spent on bribes conditional on being certain that the proposal passes.

When the voting subgame has multiple equilibria, we assume committee members play one of the equilibria where the proposal is accepted with the smallest probability. This assumption is in the spirit of Winter (2004) and Genicot and Ray (2006): first, it rules out equilibria where the proposal passes with arbitrarily small bribes because a supermajority of non-pivotal members accept. Second, it follows naturally if a vote buyer with a large valuation for the proposal is uncertain about which equilibrium will be played. Third, it selects the equilibrium preferred by committee members.

To recap the game's timing, the vote buyer moves first and proposes a bribing scheme $(b_1, ..., b_n)$. Then, committee members learn the bribing scheme, privately observe their type, and simultaneously choose whether to vote in favor or against the proposal. Finally, the proposal is implemented if at least m members vote for it.

2.1 Symmetric Voting Strategies and Bribes

This section focuses on symmetric bribing schemes: the vote buyer bribes k members who all receive the same bribe b. For committee member i, a strategy σ_i is a mapping from disutility v_i into a probability of voting in favor. For now, we focus on symmetric equilibria, i.e., equilibria in which bribed members play the same strategy.

We first solve the voting stage. We only consider members to whom the vote buyer proposes a bribe; unbribed members are assumed to use their weakly dominant strategy and vote against the proposal. Given a bribing scheme (b, k), if a member is not pivotal, the payoff difference between voting in favor and voting against is the value of the bribe. If his vote is pivotal, a vote in favor makes the proposal pass and causes an additional disutility v_i. Denoting by π_i the pivotal probability of committee member i, he accepts the bribe and votes for the proposal if and only if

$$b \geq v_i \pi_i.$$

Thus, equilibrium strategies take a cutoff form. As we focus on symmetric equilibria, any member votes for the proposal if his type is larger than some cutoff $\overline{v}$ determined in equilibrium. Our first lemma characterizes the equilibria of the voting subgame:

Lemma 1. *In any symmetric equilibrium either*

1. *all bribed members vote in favor of the proposal, or*
2. *bribed members vote for the proposal if and only if their type is smaller than a cutoff $\overline{v}$ that satisfies $b = \overline{v}\pi(\overline{v})$ where*

$$\pi(v) = \binom{k-1}{m-1} F(v)^{m-1}(1-F(v))^{k-m}.$$

If $k = m$, $v\pi(v)$ is increasing in v, and members are pivotal with probability one at the upper bound of the distribution of types (provided that the distribution has finite support). As long as b is smaller than this upper bound, Lemma 1.2 characterizes the unique equilibrium and the proposal is rejected with positive probability. For larger values of b, the strategy profile described in Lemma 1.1 is the unique equilibrium, and the proposal is always accepted.

When $k > m$, the equilibrium where all members vote for the proposal (Lemma 1.1) always exists. Equilibria described in Lemma 1.2 are illustrated in Figure 1. To make progress in characterizing these equilibria, we assume that $v\pi(v)$ is single-peaked. As established in Technical Lemma A.1 in the appendix, a sufficient condition is $F(\cdot)$ being continuously differentiable and having an increasing generalized hazard rate:

$$\frac{\partial}{\partial v}\left(\frac{vF'(v)}{1-F(v)}\right) \geq 0.$$

This assumption is encountered in other models (Lariviere, 2006) and is satisfied by all Uniform, Normal, Lognormal, and Beta distributions. By the intermediate value theorem, the equation $\overline{v}\pi(\overline{v}) = b$ admits two solutions if $b < \underset{v}{max}(v\pi(v))$, one if $b = \underset{v}{max}(v\pi(v))$ and none otherwise. If we let $v^* := \arg\max_v v\pi(v)$, the smallest bribe such that the proposal is accepted for sure in any symmetric equilibrium is $b_k^* = v^*\pi(v^*)$.[2] For the introductory example and hence in Figure 1, we have $b_{k=3}^* = \frac{8}{27}$.

[2]More precisely, we have certain acceptance for the "smallest number above b_k^*",

We now turn to the problem of the vote buyer. Conditional on bribing k members, she needs to propose b_k^* to make the proposal pass with certainty. The resulting cost of capture is

$$k \times b_k^*.$$

We want to determine the number of bribes that minimizes this cost. While bribing an additional member requires paying one more bribe, it also helps convince bribed members that they are not pivotal. Hence, it decreases b_k^*. Which effect dominates depends on the distribution of types. Key in the analysis is the dispersion of the distribution, which represents the uncertainty about committee members' disutility and determines the pivotal probabilities. We use the definition of dispersion from Shaked and Shanthikumar (2007, p.213):

Definition 1. *We say that* $\tilde{F}(\cdot)$ *is* more dispersed *than* $F(\cdot)$ *if* $\tilde{F} \leq_* F$, *i.e. if the ratio of the inverse CDFs,* $\frac{\tilde{F}^{-1}(p)}{F^{-1}(p)}$, *is nondecreasing in* p.

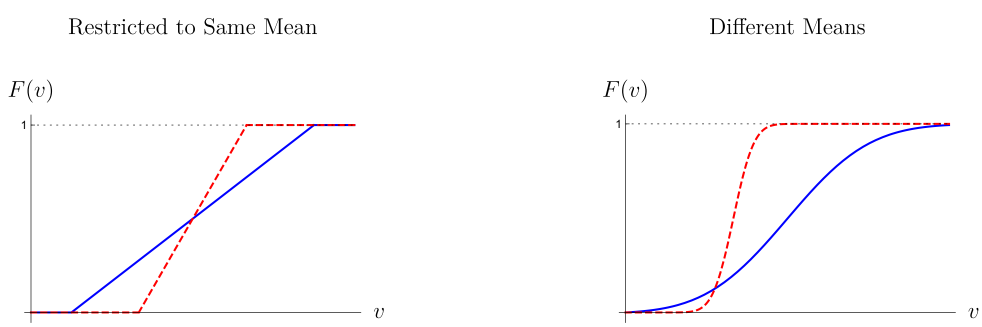

Figure 2: Dispersion Comparison

Notes: Dispersion ranking with $F(v)$ red and dashed, and the more dispersed $\tilde{F}(v)$ blue. Left panel illustrates two dispersion ranked CDFs with same mean; right panel shows CDFs with different means.

We exhibit examples of distributions that are dispersion ranked in Figure 2. For instance, uniform distributions centered around 1/2: $U[\frac{1}{2}-\alpha, \frac{1}{2}+\alpha]$ are more dispersed as α increases. The right panel shows that some distributions with different means can also be dispersion ranked.

which is not defined because bribes are on a continuum, but makes sense as the limit of a grid.

The dispersion affects the number of bribes offered by the vote buyer. To see this, recall that $b_k^* = \underset{v}{max}(v\pi(v))$ and consider the function $v\pi(v)$: the linear component, v, is independent of k and of the dispersion. It captures the fact that, for a given pivotal probability, larger types are more likely to vote against the proposal. The second component, $\pi(v)$ is affected by an increase in k through two channels. First, the value of the maximal pivotal probability decreases. Second, the largest pivotal probabilities are obtained for smaller values of v: For example, the maximum of $\pi(v)$ is reached for $v = F^{-1}(\frac{m-1}{k-1})$. For $v_i \sim U[\frac{1}{2} - \alpha, \frac{1}{2} + \alpha]$, $F^{-1}(p) = 1/2 - \alpha + 2\alpha p$. Therefore,

$$\partial F^{-1}\Big(\frac{m-1}{k-1}\Big)/\partial k = -2\alpha\Big(\frac{m-1}{(k-1)^2}\Big) < 0$$

and the maximum of $\pi(v)$ is indeed reached for a smaller v when k increases. This second point matters for $v\pi(v)$ through the complementarity between v and $\pi(v)$. When large pivotal probabilities can be sustained for high values of v, large bribes are needed to make the proposal pass. Thus, b_k^* decreases with k because it dissociates large values of v from large pivotal probabilities. Dispersion affects this channel: in response to an increase in k, the pivotal probability shifts more to the left when the distribution is more dispersed. In the uniform example, the shift of the maximum $\partial F^{-1}(\frac{m-1}{k-1})/\partial k$ is amplified by the dispersion parameter α. It implies that, for dispersed distributions, the negative effect of an additional bribe on the capture cost is larger. Thus, the vote buyer's incentive to bribe an additional member is larger when the distribution is more dispersed, and we have:

Proposition 1. *The equilibrium number of bribes k^* is increasing in the dispersion of the distribution of types $F(\cdot)$.*

We illustrate this result in Figure 3a. We simulate the equilibrium number of bribes k^* for $U[\frac{1}{2} - \alpha, \frac{1}{2} + \alpha]$ and observe that it increases with dispersion α. Even when the distribution converges to a single mass point ($\alpha = 0$), the vote buyer bribes a substantial supermajority of $\frac{3}{2}m$. In such a case, all members have the same disutility. In an equilibrium with symmetric strategies, each (bribed) member votes for with probability p. Thus, $b_k^* = max_p \pi(p)$, where $\pi(p)$ is maximized for $p^* = \frac{m-1}{k-1}$. The

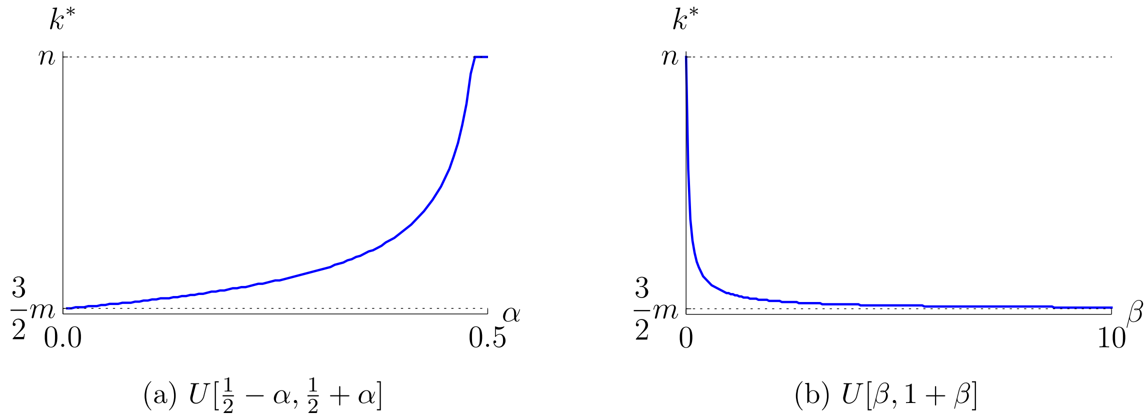

Figure 3: The Number of Members Bribed is Increasing in Dispersion.

Notes: We depict k^*, the equilibrium number of bribes, as we vary the dispersion of uniform distributions. Both panels assume $(n,m)=(300,50)$.

general F.O.C. for the equilibrium number of bribes can be rewritten as

$$\frac{\partial(kb_k^*)}{\partial k}=0 \quad\Leftrightarrow\quad \underbrace{\frac{\partial b_k^*}{\partial k}\frac{k}{b_k^*}}_{:=\epsilon_k}=-1.$$

At k^*, the elasticity of b_k^* w.r.t. k is -1. By any standard central limit theorem, $\text{Binomial}(n,p)\xrightarrow{d}\text{N}\Big(np,np(1-p)\Big)$. Thus, when members vote for with probability p^*, the distribution of votes in favor among $k-1$ other members follows $\text{Binomial}(k-1,p^*)$ and can be approximated by $\text{N}\Big((k-1)p^*,(k-1)p^*(1-p^*)\Big)$. The relevant elasticity is then given by

$$\epsilon_k=\frac{\partial\log}{\partial\log k}\underbrace{\left(\frac{\phi(0)}{\sqrt{(k-1)p^*(1-p^*)}}\right)}_{=b_k^*}=-\frac{1}{2}p^*\frac{k}{k-m}.$$

Moreover, as $p^*=\frac{m-1}{k-1}\approx\frac{m}{k}$ for large m, $\epsilon_k\approx-\frac{1}{2}\frac{m}{k-m}$ and $\epsilon_k=-1$ yields $k^*\approx\frac{3}{2}m$. In the appendix, we verify that this heuristic result also applies exactly for small m, i.e. when we consider the true, Binomial distribution of votes.

By definition, all distributions are more dispersed than a single mass point. Thus, given the just preceding discussion, Proposition 1 implies that $\frac{3}{2}m$ (or the largest integer below) is a lower bound for the equilibrium number of bribes. We now consider the number of bribes for dispersed distributions. Figure 3a suggests the vote buyer bribes all members for α sufficiently large. The number of bribes is eventually constrained by the committee's size, as we must have $k \leq n$. However, if $\alpha = 1/2$, the result holds independently of n, and the vote buyer offers as many bribes as possible. To see this, first notice that $\alpha = 1/2$ implies $v_i \sim U[0,1]$, $F(v) = v$ and

$$\begin{aligned} v\pi(v) &= \binom{k-1}{m-1} v^m (1-v)^{k-m} \\ &= \frac{m}{k}\binom{k}{m} v^m (1-v)^{k-m} \\ &= \frac{m}{k}\mathbb{P}\Big(m \text{ votes in favor} \mid k \text{ members have cutoff } v\Big). \end{aligned}$$

Thus, $v\pi(v)$ is proportional to the probability that exactly m of k bribed members vote in favor. Given that the number of votes for is distributed as Binomial(k, v), this probability is maximized for $v^* = \frac{m}{k}$. For v^*, the mean and mode of the distribution are both m. Therefore, capture cost is

$$kb_k^* = m\mathbb{P}\Big(m \text{ votes in favor} \mid k \text{ members have cutoff } \frac{m}{k}\Big).$$

In the voting profile determining the capture cost when k members are bribed, the number of votes in favor is distributed as Binomial$(k, \frac{m}{k})$. The cheapest cost of capture is achieved when k minimizes the probability of exactly m votes in favor. The mean of Binomial$(k, \frac{m}{k})$ is $k \times \frac{m}{k} = m$ and does not depend on k. However, k does increase the variance of this distribution, which is $k \times \frac{m}{k}\big(1 - \frac{m}{k}\big) = m\big(1 - \frac{m}{k}\big)$. Heuristically, as the number of trials of a Binomial increases for a mean held constant, the spread of the distribution of realizations increases, and the probability of obtaining exactly the mean decreases. Hence, as we formally prove in the appendix, the vote buyer chooses as large a k as feasible to maximize the variance of the distribution of votes: this minimizes the probability of exactly m votes for the proposal, which, in turn, minimizes capture cost.

We focused on uniform distributions with a mean of 1/2, but our results do not depend on this value. More generally, if $v_i = \eta \times z_i,\ z_i \sim F(\cdot),\ \eta \in \mathbb{R}^+$, the equilibrium number of bribes is independent of η: v^* as well as b_k^* are proportional to η. Thus, η does not affect the equilibrium number of bribes. As a result, the vote buyer bribes all members for $v_i \sim [0, \eta]$. Instead, Figure 3b considers shifts in the uniform distribution, holding the size of the support constant. Moving the uniform support to the right on the real line decreases the variance relative to the mean, which results in less dispersion. We observe the same limits for the equilibrium number of bribes as in Figure 3a. The next proposition summarizes these findings while accounting for the integer constraint on the number of bribes,

Proposition 2. *The smallest equilibrium number of bribes is* $k^* \in \left[\frac{3}{2}m - \frac{1}{2}, \frac{3}{2}m + \frac{1}{2}\right]$ *and is chosen when* $v_i \equiv v \in \mathbb{R}$. *Moreover, all members are bribed if the distribution is at least as dispersed as* $U[0,1]$.

2.2 Asymmetric Voting Strategies

Can members reject the bribes derived in Section 2.1 with asymmetric strategies? Example 1 shows that the focus on symmetric strategies is not always without loss of generality, but for sufficiently dispersed distributions, all members accepting is the unique equilibrium of the voting stage.

Example 1. Consider a distribution with a single mass point, i.e. $v_i \equiv v \in \mathbb{R}$. With symmetric strategies, we established that the optimal number of bribes is close to $\frac{3}{2}m$. However, these bribes are smaller than v; thus, any (asymmetric) strategy profile where exactly $m-1$ members accept the bribe is an equilibrium: others are pivotal and prefer to reject. Indeed, with no dispersion, the cheapest bribing scheme such that the proposal is rejected in all equilibria is $b = v$ offered to $m-1$ voters.

For sufficiently dispersed distributions, however, there are no asymmetric equilibria when members face the bribes of Section 2.1. Intuitively, dispersion makes the behavior of other members harder to predict, which prevents coordination on asymmetric equilibria. The proof relies on an iterated deletion of strictly dominated strategies. Member i's pivotal probability is maximized if others split suitably between two extreme cutoffs. In particular, if $m-1$ other (bribed) members always accept (cutoff at 1) and $k-m+1$ always reject (cutoff at 0), member i is pivotal for sure. Even when he thinks he is pivotal, member i still votes in favor if $v_i < b_k^*$. Hence, cutoffs below b_k^* are not rationalizable. Once those strategies have been eliminated, member i cannot anticipate being pivotal with certainty, and a second set of cutoffs is not rationalizable. For distributions more dispersed than $U[0,1]$, no strategy is eventually rationalizable, and the proposal is accepted in all equilibria:

Proposition 3. *If the distribution is at least as dispersed as $U[0,1]$, offering b_k^* to k members ensures the proposal is accepted with certainty in any equilibrium.*

2.3 Asymmetric Bribes

Can asymmetric bribes reduce the cost of capture? Example 2 illustrates that the vote buyer can 'divide and conquer' when the distribution is very dispersed.

Example 2. Let $(n, m) = (3, 2)$ and $v_i \sim \text{Bernoulli}(\frac{1}{2})$. Suppose the vote buyer offers the same bribe to all 3 members. A type $v_i = 0$ accepts, implying that the pivotal probability is maximized if $v_i = 1$ vote against. Then, $\pi = \frac{1}{2}$, which is the value of the bribe needed to make the proposal pass. Capture costs $\frac{3}{2}$. The distribution being more dispersed than $U[0,1]$, there is no asymmetric equilibrium (Proposition 3).

Now, consider the asymmetric bribes $(b_1, b_2, b_3) = (\frac{1}{2}, \frac{1}{2}, \epsilon)$. A member with $v_i = 0$ accepts, and the pivotal probability of any member cannot exceed $\frac{1}{2}$. Thus, voters 1 and 2 accept. In turn, voter 3 is not pivotal and accepts regardless of his type. The capture cost is 1, and the vote buyer pays less with asymmetric bribes.

However, symmetric bribes can be preferred when the distribution is less dispersed. In particular, the solution of offering $\frac{8}{27}$ to all members as in our introductory example generalizes:

Example 3. Consider $(n, m) = (3, 2)$ and $v_i \sim U[0,1]$. Denoting by $\overline{v}_i$ the equilibrium cutoff of member i, an equilibrium where all cutoffs are interior satisfies:

$$\begin{aligned}
\overline{v}_1 \pi_1(\overline{v}_2, \overline{v}_3) &= b_1, \\
\overline{v}_2 \pi_2(\overline{v}_1, \overline{v}_3) &= b_2, \\
\overline{v}_3 \pi_3(\overline{v}_1, \overline{v}_2) &= b_3.
\end{aligned} \tag{1}$$

When bribes are large enough, such an equilibrium does not exist and all committee members voting in favor is the unique equilibrium. Thus, the vote buyer proposes the cheapest (b_1, b_2, b_3) such that (1) has no solution.

To identify this bribing scheme, it is useful to look at the Jacobian of (1):

$$J = \begin{pmatrix} \overline{v}_2(1-\overline{v}_3) + (1-\overline{v}_2)\overline{v}_3 & \overline{v}_1(1-2\overline{v}_3) & \overline{v}_1(1-2\overline{v}_2) \\ \overline{v}_2(1-2\overline{v}_3) & \overline{v}_1(1-\overline{v}_3) + (1-\overline{v}_1)v_3 & (1-2\overline{v}_1)\overline{v}_2 \\ (1-2\overline{v}_2)\overline{v}_3 & (1-2\overline{v}_1)\overline{v}_3 & \overline{v}_1(1-\overline{v}_2) + (1-\overline{v}_1)\overline{v}_2 \end{pmatrix}$$

For given bribes (b_1, b_2, b_3), suppose there exist $(\overline{v}_1, \overline{v}_2, \overline{v}_3)$ solving (1). As long as J is non-singular, following any ϵ-perturbation of (b_1, b_2, b_3) there also exists a solution to (1). However, when the determinant is 0, we can find an ϵ-perturbation of the bribes such that (1) has no solution, which guarantees certain passing. Moreover cost cannot be minimized if some members receive more than what is exactly needed to make the proposal pass. Therefore, the solution to the vote buyer's problem is arbitrarily close to the cheapest combination of bribes for which the determinant of the Jacobian is 0.

Computing the determinant of the Jacobian matrix gives $2\overline{v}_1\overline{v}_2\overline{v}_3(2-\overline{v}_1-\overline{v}_2-\overline{v}_3)$, i.e. the matrix is singular if[3] $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 = 2$. Thus, if a combination of bribes is supported by an equilibrium satisfying $\sum \overline{v}_i = 2$, then an arbitrarily close combination ensures that the proposal always passes. As a result, we have to find $(\overline{v}_1, \overline{v}_2, \overline{v}_3) \in [0,1]^3, \sum \overline{v}_i = 2$ that minimizes $\sum b_i$. Without loss of generality, suppose $\overline{v}_1 < \overline{v}_2 < \overline{v}_3$. We now show that decreasing the distance between cutoffs decreases $\sum b_i$. Note

$$\sum b_i = \sum \overline{v}_i \pi_i = m \times \mathbb{P}(m \text{ votes in favor} \mid \overline{v}_1, \overline{v}_2, \overline{v}_3),$$

where of course $m = 2$. The number of votes in favor becomes more uncertain when the cutoffs become closer to each other. This lowers the probability mass at the mean of the distribution of votes in favor. In particular, let us increase $\overline{v}_1$ and decrease $\overline{v}_3$ by the same amount: $d\overline{v}_1 = 1$, $d\overline{v}_3 = -1$ and $d\overline{v}_2 = 0$, which keeps $\sum \overline{v}_i$ constant.

[3] The determinant is also 0 if some cutoffs are 0. As $\overline{v}_i \leq b_i$, $\overline{v}_i = 0$ can only be part of an equilibrium if $b_i = 0$. But then, the proposal passes with probability 1 if the two other members receive a bribe of 1, which does not minimize the capture cost.

Using the Jacobian, the impact on the capture cost is

$$\sum_i db_i = (\overline{v}_1 - \overline{v}_3)(6\overline{v}_2 - 2),$$

which is negative because $\overline{v}_1 < \overline{v}_3$ and[4] $\overline{v}_2 > 1/3$. As a result, under the constraint $\sum \overline{v}_i = 2$, $\overline{v}_1 = \overline{v}_2 = \overline{v}_3 = \frac{2}{3}$ minimizes $\sum b_i$. These cutoffs are an equilibrium if $b_i = \frac{8}{27}$ for all members. Thus, the symmetric bribing scheme identified in the introductory example does minimize capture cost.

3 Sequential Voting

We now consider a committee voting sequentially. We suppose the vote buyer knows the order of members during the vote, which is, for instance, the case in the US Senate where members vote in alphabetical order. In this section, bribes can be of different values and we assume $v_i \sim U[0, 1]$.

In the voting stage, members use backward induction to infer their pivotal probability as in Spenkuch et al. (2018). Define $S(x, y)$ as the subgame where x votes are needed to pass the proposal, and y other members vote afterward. If in $S(x, y)$ the member votes for the proposal, $S(x - 1, y - 1)$ is reached while a vote against leads to $S(x, y - 1)$. As the vote buyer can condition bribes on the order of members, bribes b_y are now indexed by $y \in \{0, ..., n - 1\}$, the number of members still to vote.[5]

Let $v(x, y)$ be the cutoff played in $S(x, y)$ and $p(x, y)$ be the probability that the proposal is accepted given that $S(x, y)$ is reached. We jointly characterize $v(x, y)$ and $p(x, y)$ to find the equilibrium of the game. We provide an example of an arbitrary bribing scheme in Table 1, where we focus on the last four members. We display $v(x, y)$ in the left panel of the table and $p(x, y)$ in the right panel. When a member votes for the proposal, the subgame located North-West along the diagonal is reached;

[4]As $\overline{v}_1 + \overline{v}_2 + \overline{v}_3 = 2$ and $\overline{v}_3 < 1$, we obtain $\overline{v}_1 + \overline{v}_2 \geq 1$. Combining with $\overline{v}_2 > \overline{v}_1$, we must have $\overline{v}_2 > 1/3$.

[5]If bribes can jointly depend on x and y, capture is costless. The vote buyer can offer arbitrarily small bribes in subgames $S(m - s, n - 1 - s)$ for $s \in \{0, ..., m - 1\}$ and $b(x, y) = 1$ otherwise. The proposal is always accepted, so all members vote in favor and get ϵ. As in Dal Bo (2007), the subgames where bribes are 1 are never reached.

if he votes against it, we move to the subgame just to the North. We solve the game backward and start with the last member, $y = 0$. If $x < 1$, the proposal is accepted regardless of the vote of the last member and, provided that the bribe is positive, he accepts: $v(x, 0) = p(x, 0) = 1$. When $x = 1$, the last member supports the proposal if $b_0 > v_i$ and we have $v(1, 0) = p(1, 0) = b_0$. Finally, if $x > 1$, the proposal will be rejected and $v(x, 0) = 1$, $p(x, 0) = 0$.

We now characterize the equilibrium strategy for all members. When $x \leq 0$, the proposal is already accepted and we have $p(x, y) = v(x, y) = 1$. Moreover, if $x > y+1$, the proposal would be rejected even if member y as well as all the following members voted for it, which implies $p(x, y) = 0$ and $v(x, y) = 1$. Now, consider the subgames where $0 < x \leq y + 1$. Suppose we know the strategies played by members of rank $i \leq y - 1$. If member y votes in favor, his expected utility is $b_y - v_y p(x - 1, y - 1)$ while a vote against gives $-v_y p(x, y - 1)$. Therefore, in $S(x, y)$, the member votes in favor of the proposal if his type is larger than a cutoff $v(x, y)$ defined by:

$$v(x, y) = \min\left\{\frac{b_y}{p(x-1, y-1) - p(x, y-1)}, 1\right\}.$$

As a result, the probability that the proposal passes given that $S(x, y)$ is reached is:

$$p(x, y) = v(x, y)p(x - 1, y - 1) + (1 - v(x, y))p(x, y - 1).$$

	$v(x,y)$				$p(x,y)$			
	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$y = 0$	1	b_0	1	1	1	b_0	0	0
$y = 1$	1	$\frac{b_1}{1-b_0}$	$\frac{b_1}{b_0}$	1	1	$b_0 + b_1$	b_1	0
$y = 2$	1	$\frac{b_2}{1-(b_0+b_1)}$	1	1	1	$b_0 + b_1 + b_2$	$b_0 + b_1$	b_1
$y = 3$	1	$\frac{b_3}{1-(b_0+b_1+b_2)}$	$\frac{b_3}{b_2}$	$\frac{b_3}{b_0}$	1	$b_0 + b_1 + b_2 + b_3$	$b_0 + b_1 + b_3$	$b_1 + b_3$

Table 1: Equilibrium of the voting stage, an example.

Notes: Equilibrium cutoffs (left panel) and acceptance probability (right panel) for $b_2 > b_0 > b_1 > b_3$, $\sum_{s=0}^{3} b_s < 1$. x is the number of votes required to pass the proposal, y the number of members still to vote.

Using these equations, we first derive the probabilities of acceptance at each subgame in which there is just one vote in favor needed to pass the proposal:

Lemma 2. $p(1, y) = \min\Big\{ \sum_{s=0}^{y} b_s, 1\Big\}$.

We proceed by induction to establish this lemma. Recall that for all y, $p(0, y-1) = 1$. Using the definition of $v(1, y)$, this implies:

$$v(1, y) = \min\Big\{\frac{b_y}{1 - p(1, y-1)}, 1\Big\}.$$

Plugging this expression in the definition of $p(1, y)$ and rearranging, we get $p(1, y) = \min\Big\{b_y + p(1, y-1), 1\Big\}$. To complete the proof, notice that the Lemma holds for $y = 1$ and assume that it holds for $y-1$. Then $p(1, y-1) = \min\Big\{ \sum_{s=0}^{y-1} b_s, 1\Big\}$ and we do have $p(1, y) = \min\Big\{ \sum_{s=0}^{y} b_s, 1\Big\}$, which proves the claim.

Taking Lemma 2 as base case, we use an induction (on y) within an induction (on x) to find:[6]

Lemma 3. *Let* $\bar{b}_{y,1} = \max\{b_0, \ldots, b_y\}$ *and* $\bar{b}_{y,s} := \max\Big(\{b_0, \ldots, b_y\}\backslash\{\bar{b}_{y,1}, \ldots, \bar{b}_{y,s-1}\}\Big)$ *for* $s > 1$*. Then for* $x \geq 1$*, we have*

$$p(x, y) = \min\Big\{ \sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1} \bar{b}_{y,s}, 1\Big\}.$$

In words, if the probability of acceptance in $S(x, y)$ is smaller than 1, it is the sum of the $y+1$ remaining bribes after subtracting the $x-1$ highest bribes. Intuitively, the members who receive the largest bribes accept regardless of their type and rely on others to reject the proposal: in $S(x, y)$, if b_y is one of the $x-1$ largest bribes among members still to vote, member y votes in favor: $v(x, y) = 1$. Instead, if b_y is one of the smallest bribes, the problem of member y is equivalent to the one described in Lemma 2: given that at least $x-1$ of the remaining members will accept, he is virtually in a committee with $x = 1$ in which the following members have received the remaining $y - x + 1$ smallest bribes. Therefore, the probability of acceptance of the proposal is as discussed in Lemma 2.

[6]Note that this Lemma uses the convention that $\sum_{s=1}^{x-1} \bar{b}_{y,s} = 0$ if $x = 1$.

We can now consider the problem of the vote buyer. Given that the voting stage starts in subgame $S(m, n-1)$, the vote buyer chooses the cheapest combination $\{b_y\}_{y=0}^{n-1}$ such that $p(m, n-1) = 1$. Using Lemma 3, we can write this problem as

$$\min_{\{b_y\}_{y=0}^{n-1}} \sum_{y=0}^{n-1} b_y \quad \text{s.t.} \quad \sum_{s=0}^{n-1} b_s - \sum_{s=1}^{m-1} \overline{b}_{n-1,s} \geq 1.$$

We have established that the proposal passes with probability 1 in $S(m, n-1)$ if the sum of the $n-(m-1)$ smallest bribes is at least 1. The $m-1$ largest bribes do not affect the probability of acceptance and, as long as they are larger than the m^{th} highest bribe $\overline{b}_{n-1,m}$, lowering them decreases the vote buyer's expenditure without affecting the passing of the proposal. As a result, the cost is minimized when the $m-1$ largest bribes are equal to $\overline{b}_{n-1,m}$. Moreover, this value is the maximum of the $n-(m-1)$ smallest bribes. Given that the sum of these bribes is 1, the smallest $\overline{b}_{n-1,m}$ is achieved when all the $n-(m-1)$ smallest bribes are equal to $\frac{1}{n-m+1}$. Therefore, even the member who receives the lowest bribe gets $\frac{1}{n-m+1}$, which implies that all members are bribed. Moreover, the $m-1$ largest bribes are also equal to the $n-(m-1)$ smallest bribes, and we conclude that all bribes are equal:

Proposition 4. *When voting is sequential, the vote buyer bribes all members equally and offers them* $b = \frac{1}{n-m+1}$.

Hence, the primary mechanism of the paper also applies to sequential voting: the vote buyer bribes a supermajority to make a pivotal event unlikely. By doing so, she makes sure that the committee supports the proposal at the lowest possible cost.

4 Cost of Capture

We now consider the effect of committee design on the capture cost and focus on $v_i \sim U[0,1]$ to compare simultaneous and sequential voting. The cost to the vote buyer in the two cases is given by

$$C^{sim}(n,m) = n\binom{n-1}{m-1}\left(\frac{m}{n}\right)^{m}\left(1-\frac{m}{n}\right)^{n-m},$$
$$C^{seq}(n,m) = \frac{n}{n-m+1}.$$

In both cases, the vote buyer bribes as many members as possible but is constrained by the number of members n. Adding members without changing the majority requirement relaxes this constraint and strictly decreases the cost. Conversely, raising m without changing n strictly increases the cost: manipulating pivotal considerations is more challenging with a more demanding majority requirement. If we require unanimity to pass the proposal ($m = n$), the vote buyer cannot do better than bribing the minimal winning coalition and pay the maximal possible disutility to m members. Unanimity uniquely protects the committee from outside influence by severing the pivotal channel.

Moreover, suppose we multiply both n and m by the same factor, which keeps the majority rule $\frac{m}{n}$ fixed. Unless $m = n$, the cost function is not homogeneous of degree one and increases less than proportionally because members expect to be pivotal with a lower probability in a larger committee. Thus, we have:

Proposition 5. *If $C(\cdot)$ is the capture cost (under either simultaneous or sequential voting), $\partial C/\partial n < 0$, $\partial C/\partial m > 0$. Furthermore, if $m < n$ and $\lambda > 1$, $C(\lambda n, \lambda m) < \lambda C(n, m)$.*

Does sequential or simultaneous voting maximize cost? The answer depends on the size of the committee. Consider a scalar λ multiplying both n and m. With sequential voting, the capture cost is determined by the fact that the sum of the $n - m + 1$ smallest bribes must be 1. This requirement provides a lower bound on the capture cost: even in a small committee, the cost cannot be smaller than 1. Instead, the vote buyer can pay less than 1 for a small committee with simultaneous voting, as we have seen in the introductory example. Indeed, we have:

$$\lim_{\lambda\to 0}\frac{C^{seq}(\lambda n,\lambda m)}{\lambda} = 1 > \frac{m}{n} = \lim_{\lambda\to 0}\frac{C^{sim}(\lambda n,\lambda m)}{\lambda}.$$

Therefore, the capture cost is higher with sequential voting when the committee is small. However, the fact that the sum of $n - m + 1$ bribes is 1 also implies an upper bound for the cost with sequential voting. As all bribes are eventually equal, the cost is 1 divided by the share represented by these $n - m + 1$ bribes. For example, if $(n, m) = (3, 2)$, the sum of two bribes must be equal to 1, and their share among the three bribes offered is $2/3$, which implies that the capture cost is $3/2$. As λ varies, the share of the $n - m + 1$ bribes is

$$\frac{\lambda(n-m)+1}{\lambda n}.$$

This share decreases with λ, but it cannot be less than $1 - \frac{m}{n}$, which implies that the cost is bounded. Instead, the cost for simultaneous voting grows unbounded and we have:

$$\lim_{\lambda\to\infty} C^{seq}(\lambda n, \lambda m) = \frac{1}{1-\frac{m}{n}} < \infty = \lim_{\lambda\to\infty} C^{sim}(\lambda n, \lambda m).$$

We conclude that capture cost is higher with sequential voting in small committees, while the opposite is true in large committees:

Proposition 6. *If $m < n$, there exists a λ^* such that the capture cost is larger under simultaneous voting $C^{sim}(\lambda n, \lambda m) > C^{seq}(\lambda n, \lambda m)$ if $\lambda > \lambda^*$.*

5 Concluding Remarks

When voters have uncertain preferences, we have established that the vote buyer captures supermajorities to manipulate pivotal considerations. We only considered payments conditioned on individual voting decisions. We conclude with a discussion of other contractual environments.

If bribes are conditioned on the passing of the proposal, a pivotal vote also decides the payment of the bribes. Thus, it is a weakly dominant strategy to vote against if individual disutility exceeds the bribe. The vote buyer cannot manipulate pivotal considerations with such contracts: to make the proposal pass with certainty, she has to make an offer equal to the maximal disutility to m members. Instead, suppose bribes depend on the number of votes in favor. A vote matters for the bribe even when it is not pivotal for the passing of the proposal. In a previous version of the paper

(Louis-Sidois and Musolff 2020), we proposed an example where the vote buyer bribed a supermajority, and the proposal was always rejected due to pivotal considerations.

Furthermore, in an unrestricted contractual environment, bribes can be contingent on the whole vector of votes. In such a case, Dal Bo (2007) has established that capture occurs at no cost: the vote buyer promises a bribe exceeding the maximal possible disutility if a member is pivotal and an arbitrarily small amount otherwise. Voting in favor is then a dominant strategy, and when more than m members receive such offers, the proposal always passes. If such contracts are allowed, our solution is still relevant for a budget-constrained vote buyer: even if players are never pivotal in equilibrium, the vote buyer must be able to pay the large pivotal bribes for Dal Bo's strategy to be credible. Therefore, while our solution is more expensive (as the vote buyer actually pays the bribes), it would nevertheless be feasible for lower budget constraints.

We have considered offers visible by all. If offers are privately communicated to each member, a minimal winning coalition is bribed in equilibrium. Each bribe is equal to the maximal possible disutility, and capture is more costly with private offers. To see why bribing a supermajority is not credible, consider a voting profile where more than m members vote in favor with certainty. The vote buyer would deviate and propose exactly m bribes. This deviation cannot be detected by committee members who continue receiving the bribe, but, in equilibrium, a bribed member cannot believe there are more than $m-1$ other bribes. For him to always vote for, the bribe must be equal to the maximal possible disutility.

Finally, our model can be reinterpreted with punishments for members who vote against instead of bribes. The equilibrium of the voting stage is identical. For the vote buyer, enforcing punishment is likely to be costly, which implies she effectively pays for members who vote against the proposal. If she has enough punishment power, she uses the strategy of the paper. Capture is costless because all approached members vote in favor. The cost of capture we computed corresponds to the minimum punishment power needed to secure certain passing of the proposal.

Appendix A: Proofs

Before proceeding to the proofs, we establish some necessary prerequisites. Recall $\Gamma(\cdot)$ is a continuous extension of the factorial function. In particular, $\Gamma(x) = (x-1)!$ for $x \in \mathbb{N}$. Thus,

$$\binom{k}{m} = \exp\Big(\ln\Gamma(k+1) - \ln\Gamma(m+1) - \ln\Gamma(k-m+1)\Big).$$

The digamma function is defined as $\psi(x) = \frac{\partial}{\partial x}\ln\Gamma(x)$. Hence,

$$\frac{\partial \ln\binom{k}{m}}{\partial k} = \psi(k+1) - \psi(k+1-m), \quad \frac{\partial \ln\binom{k}{m}}{\partial m} = \psi(k+1-m) - \psi(m+1).$$

To characterize these derivatives, our proofs below will make use of the fact that $\psi(x+1) = \psi(x) + 1/x$ and hence $\psi(b) - \psi(a) = \sum_{c=a}^{b-1}(1/c)$ for $b > a$. For this and more facts about $\psi(\cdot)$, see (Abramowitz, 1972, p.258).

Lemma A.1. *Suppose $F(\cdot)$ has increasing generalized failure rate. Then $\overline{v}\pi(\overline{v})$ is single-peaked.*

Proof. Writing out the pivotal probability, we want to show the single-peakedness of

$$v\binom{k-1}{m-1}F(v)^{m-1}[1-F(v)]^{k-m},$$

with associated FOC

$$\frac{[F(v^*)-1]F(v^*)}{v^*F'(v^*)} = (m-1) - (k-1)F(v^*). \tag{2}$$

We can rewrite (2) in terms of $x = F(v)$ to get

$$\frac{x-1}{F'(F^{-1}(x))F^{-1}(x)} = \frac{m-1}{x} - (k-1). \tag{3}$$

The RHS is strictly decreasing in x. The LHS is increasing by our assumption of increasing generalized failure rates. Thus, (2) has a unique solution. □

Lemma A.2. *Suppose $\tilde{F}(\cdot)$ and $F(\cdot)$ have increasing generalized failure rates. Suppose further that $\tilde{F}(\cdot)$ is more dispersed than $F(\cdot)$. Then $\tilde{F}(\tilde{v}^*) \geq F(v^*)$.*

Proof. Consider (3) separately for the two distribution functions. We first establish that the LHS lies lower for $\tilde{F}$ than for F:

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(\frac{F^{-1}(x)}{\tilde{F}^{-1}(x)}\right) \geq 0 \\
\therefore\quad &\frac{[F^{-1}(x)]'\tilde{F}^{-1}(x) - F^{-1}(x)[\tilde{F}^{-1}(x)]'}{[\tilde{F}^{-1}(x)]^2} \geq 0 \\
\therefore\quad &\frac{1}{F'(F^{-1}(x))}\tilde{F}^{-1}(x) \geq F^{-1}(x)\frac{1}{\tilde{F}(\tilde{F}^{-1}(x))} \\
\therefore\quad &\frac{1}{F'(F^{-1}(x))F(x)} \geq \frac{1}{\tilde{F}(\tilde{F}^{-1}(x))\tilde{F}(x)} \\
\therefore\quad &\frac{x-1}{F'(F^{-1}(x))F(x)} \geq \frac{x-1}{\tilde{F}(\tilde{F}^{-1}(x))\tilde{F}(x)}
\end{aligned}$$

Given that the LHS in (3) is increasing, it follows that $\tilde{x} = \tilde{F}(\tilde{v}^*) \geq F(v^*) = x$. □

Proposition 1. *The equilibrium number of bribes k^* is increasing in the dispersion of the distribution of types $F(\cdot)$.*

Proof. The optimal choice of k is the solution to:

$$\min_k \left\{ kv^* \binom{k-1}{m-1} F(v^*)^{m-1}[1 - F(v^*)]^{k-m} \right\}$$

where v^* is an implicit function of k. By the envelope theorem, the FOC for k^* is

$$1/k + \psi^{(0)}(k) - \psi^{(0)}(k-m+1) + log(1 - F(v^*)) \;=\; 0 \tag{4}$$

where we have multiplied by strictly positive terms to eliminate their inverse. From Lemma A.2 above, $\tilde{F}(\tilde{v}^*) \geq F(v^*)$. Thus, at the optimal k associated with $F(\cdot)$, the LHS under the more dispersed distribution $\tilde{F}(\cdot)$ is negative and increasing in k. Given that it can be verified that the cost function admits at most one local minimum, this is sufficient to conclude that the optimal k under $\tilde{F}(\cdot)$ needs to be larger. □

Proposition 2. *The smallest equilibrium number of bribes is* $k^* \in \left[\frac{3}{2}m - \frac{1}{2}, \frac{3}{2}m + \frac{1}{2}\right]$ *and is chosen when* $v_i \equiv v \in \mathbb{R}$*. Moreover, all members are bribed if the distribution is at least as dispersed as* $U[0,1]$*.*

Proof. We plug the CDFs for the mentioned distributions into (4) and establish that the equilibrium number of bribes is as described. As all distributions are more dispersed than $v_i = v\ \forall i$, Proposition 1 implies that $k^* \geq \frac{3}{2}m - \frac{1}{2}$.

1. If $v_i = v\ \forall i$, bribed agents mix in a symmetric equilibrium and vote in favor with a common probability p. The pivotal probability is:

$$\pi(p) = \binom{k-1}{m-1} p^{m-1}(1-p)^{k-m}.$$

As this function is maximized at $p^* = \frac{m-1}{k-1}$, the cost of bribing k players is

$$C(k) = k \times \binom{k-1}{m-1} \left(\frac{m-1}{k-1}\right)^{m-1} \left(\frac{k-m}{k-1}\right)^{k-m}$$

Taking the derivative of this expression w.r.t. k yields

$$\frac{\partial C}{\partial k} \propto \frac{1}{k} + [\psi^{(0)}(k) - \psi^{(0)}(k-m+1)] - [\log(k-1) - \log(k-m)] =: \xi(k,m).$$

Thus, $\xi(k^*, m) = 0$ and the sign of $\xi(k,m)$ is informative as to whether $k < k^*$ or $k > k^*$. We employ the following inequality from (Chen, 2005) to bound ξ:

$$\frac{1}{2x} - \frac{1}{12x^2} < \psi^{(0)}(x+1) - \ln(x) < \frac{1}{2x}.$$

In particular, we have $\underline{\xi}(k,m) < \xi(k,m) < \overline{\xi}(k,m)$, where

$$\begin{aligned} \underline{\xi}(k,m) &= \frac{1}{k} + \frac{1}{2(k-1)} - \frac{1}{12(k-1)^2} - \frac{1}{2(k-m)}, \\ \overline{\xi}(k,m) &= \frac{1}{k} + \frac{1}{2(k-1)} + \frac{1}{12(k-m)^2} - \frac{1}{2(k-m)}. \end{aligned}$$

Using these expressions, one can establish that for $m > 3$

$$\xi\left(\frac{3}{2}m - \frac{1}{2}, m\right) < \overline{\xi}\left(\frac{3}{2}m - \frac{1}{2}, m\right) < 0 \implies k^* \geq \frac{3}{2}m - \frac{1}{2}$$

and

$$\xi\left(\frac{3}{2}m + \frac{1}{2}, m\right) > \underline{\xi}\left(\frac{3}{2}m + \frac{1}{2}, m\right) > 0 \implies k^* \leq \frac{3}{2}m + \frac{1}{2}.$$

For $m \in \{1, 2, 3\}$, we similarly find $\xi\left(\frac{3}{2}m - \frac{1}{2}, m\right) < 0 < \xi\left(\frac{3}{2}m + \frac{1}{2}, m\right)$ by direct numerical evaluation.

2. If $v_i \sim U[0, 1]$, we have $F(v^*) = \frac{m}{k}$. Plugging into (4) yields

$$\frac{\partial(kb_k^*)}{\partial k} = kb_k^*\Big\{\log\left(\frac{k-m}{k}\right) + \psi(k) - \psi(k-m+1) + \frac{1}{k}\Big\},$$

where ψ is the Digamma function. As for any decreasing function $\sum_{s=a+1}^{b} g(s) < \int_a^b g(s)ds$,

$$\begin{aligned}\psi(k) - \psi(k-m+1) + \frac{1}{k} &= \sum_{s=1}^{m} \frac{1}{k-m+s} \\ &< \int_0^m \frac{1}{k-m+s} ds = -log\Big(\frac{k-m}{k}\Big).\end{aligned}$$

Thus, the cost of capture is strictly decreasing in k so that $k^* = n$. □

Proposition 3. *If the distribution is at least as dispersed as $U[0, 1]$, offering b_k^* to k members ensures the proposal is accepted with certainty in any equilibrium.*

Proof. Consider at no loss of generality the strategy of (bribed) member 1. To begin the iterated elimination, note that choosing any cutoff below b_k^* is a dominated strategy. If for all other bribed members i the smallest rationalizable cutoff after iteration t is ℓ_i^t, member 1's smallest rationalizable cutoff ℓ_1^{t+1} at iteration $t+1$ solves

$$\ell_1^{t+1} \times \left(\max_{\overline{v}_i \in [\ell_i^t, 1]} \pi(\overline{v}_2, \ldots, \overline{v}_k)\right) = b_k^*. \tag{5}$$

Given symmetry, we have $\ell_i^t = \ell_j^t$ for all i, j and hence refer simply to ℓ^t. The remainder of the proof shows that if $\ell^t < 1$, then $\ell^{t+1} > \ell^t + \epsilon$ (where ϵ is bounded away from 0).

1. Let $f_x(\mathbf{v}^y)$ be the probability that among all members but y there are exactly x votes in favor (writing $\mathbf{v}^y = (\overline{v}_1, \ldots, \overline{v}_{y-1}, \overline{v}_{y+1}, \ldots, \overline{v}_k)$ for the vector of cutoffs of all members other than y). For any i,

$$\pi_1(\mathbf{v}^1) = f_{m-1}(\mathbf{v}^1) = F(\overline{v}_i) f_{m-2}(\mathbf{v}^{1i}) + (1 - F(\overline{v}_i)) f_{m-1}(\mathbf{v}^{1i}),$$

whence $\frac{\partial f_{m-1}(\mathbf{v}^1)}{\partial \overline{v}_i}$ is independent of $\overline{v}_i$. Thus, there is a solution to the maximization problem in (5) in which $\overline{v}_i \in \{\ell^t, 1\}$ for all i.

2. In light of this, let $\pi_h(\ell^t)$ be the value of the pivotal probability if exactly h of the $k-1$ members choose a cutoff of $\overline{v}_i = 1$ and $k-1-h$ choose a cutoff of ℓ^t; then the value of $\max_{\overline{v}_i \in [\ell^t_i, 1]} \pi(\overline{v}_2, \ldots, \overline{v}_k)$ in (5) is $\max_h \pi_h(\ell^t)$.

3. For any h, we can derive the bribe $b^*_{k,h}$: when offered to $k-h$ members in a committee of $n-h$, $b^*_{k,h}$ ensures there is no equilibrium in which bribed members vote against the proposal with positive probability. To be exact,

$$b^*_{k,h} = max_v \{ v \underbrace{\binom{k-h-1}{m-h-1} [F(v)]^{m-1-h} [1-F(v)]^{k-m}}_{\pi_h(v)} \} + \epsilon.$$

4. We now show that $b^*_{k,h} < b^*_k$ for all $h \in \{1, ..., m\}$ (indeed $b^*_{k,0} = b^*_k$). We have:

$$\frac{\partial b^*_{k,h}}{\partial h} = -b^*_{k,h} \times \{ log[F(v^*_h)] + \psi(k-h) - \psi(m-h) \}, \qquad (6)$$

where ψ is the Digamma function. In particular, if $v_i \sim U[0,1]$,

$$\frac{\partial b^*_{k,h}}{\partial h} = -b^*_{k,h} \times \left\{ log\Big(\frac{m-h}{k-h}\Big) + \psi(k-h) - \psi(m-h) \right\}. \qquad (7)$$

where ψ is the Digamma function. This expression is negative as $\psi(x) =$

$\psi(x-1)+\frac{1}{x-1}$ and for any decreasing function $\sum_{s=a}^{b-1} g(s) > \int_a^b g(s)ds$ and hence

$$\begin{aligned}\psi(k-h)-\psi(m-h) &= \sum_{s=m-h}^{s=k-h-1} \frac{1}{s} \\ &> \int_{m-h}^{k-h} \frac{1}{s} ds = \log(k-h)-\log(m-h).\end{aligned}$$

By Lemma 3, $F(v_h^*)$ is larger for more dispersed distributions. Thus, as (7) is negative, (6) must also be negative for distributions more dispersed than $U[0,1]$.

5. Finally, note that for $\ell^t < 1$ (with index on equalities/inequalities referring to the relevant step in the proof),

$$\begin{aligned}\ell^t \times \left(\max_{\overline{v}_i \in [\ell^t,1]} \pi(\overline{v}_2,\dots,\overline{v}_k) \right) &=_{(1,2)} \quad \ell^t \max_h \pi_h(\ell^t) \\ &\leq_{(3)} \quad \max_h b_{k,h}^* - \epsilon \\ &<_{(4)} \quad b_k^* - \epsilon.\end{aligned}$$

But from (5),

$$\begin{aligned}(\ell^{t+1}-\ell^t) \times \left(\max_{\overline{v}_i \in [\ell^t,1]} \pi(\overline{v}_2,\dots,\overline{v}_k) \right) &= b_k^* - \ell^t \times \left(\max_{\overline{v}_i \in [\ell^t,1]} \pi(\overline{v}_2,\dots,\overline{v}_k) \right) \\ &> \epsilon.\end{aligned}$$

□

Lemma 3. *Let* $\overline{b}_{y,1} = \max\{b_0,\dots,b_y\}$ *and* $\overline{b}_{y,s} := \max\Big(\{b_0,\dots,b_y\}\backslash\{\overline{b}_{y,1},\dots,\overline{b}_{y,s-1}\}\Big)$ *for* $s > 1$. *Then for* $x \geq 1$, *we have*

$$p(x,y) = \min\Big\{ \sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1} \overline{b}_{y,s}, 1 \Big\}.$$

Proof. We proceed by induction on x. Lemma 2 proves the base case ($x=1$). Suppose the result holds for $x-1$. We prove that it also holds for x. To do so, we use an induction on y.

- Base Case: $y=0$. If $x \geq 2$, $p(x,0)=0$; if $x=1$ then $p(x,0)=b_0$ as required.

- Inductive Step. Rearranging the equation for a voter's cutoff and then employing the inductive hypothesis, we have

$$\begin{aligned} v(x,y) &= \min\left\{\frac{b_y}{p(x-1,y-1)-p(x,y-1)},1\right\} \\ &= \begin{cases} \min\left\{\frac{b_y}{\overline{b}_{y-1,x-1}},1\right\} & \text{if } p(x-1,y-1)<1, \\ \min\left\{\frac{b_y}{1-(\sum_{s=0}^{y-1} b_s - \sum_{s=1}^{x-1}\overline{b}_{y-1,s})},1\right\} & \text{if } p(x-1,y-1)=1 \\ & \text{and } p(x,y-1)<1, \\ 1 & \text{if } p(x,y-1)=1. \end{cases} \end{aligned}$$

We now verify the expression for $p(x,y)$ following these cases.

  - Assume $p(x-1,y-1)<1$.
    * Assume $b_y > \overline{b}_{y-1,x-1}$. Then $v(x,y)=1$ so that

$$\begin{aligned} p(x,y) &= p(x-1,y-1) \\ &= \min\left\{\sum_{s=0}^{y-1} b_s - \sum_{s=1}^{x-2}\overline{b}_{y-1,s},1\right\} \\ &= \min\left\{\sum_{s=0}^{y} b_s - \left(\sum_{s=1}^{x-2}\overline{b}_{y-1,s}+b_y\right),1\right\} \\ &= \min\left\{\sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1}\overline{b}_{y,s},1\right\}. \end{aligned}$$

    * Assume $b_y \le \overline{b}_{y-1,x-1}$. Then

$$\begin{aligned} p(x,y) &= p(x,y-1)+v(x,y)[p(x-1,y-1)-p(x,y-1)] \\ &= \sum_{s=0}^{y-1} b_s - \sum_{s=1}^{x-1}\overline{b}_{y-1,s} + \frac{b_y}{\overline{b}_{y-1,x-1}} \times \overline{b}_{y-1,x-1} \\ &= \sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1}\overline{b}_{y-1,s} = \sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1}\overline{b}_{y,s}, \end{aligned}$$

    where we obtain the last line because $b_y \le \overline{b}_{y-1,x-1}$ implies $\sum_{s=1}^{x-1}\overline{b}_{y-1,s} = \sum_{s=1}^{x-1}\overline{b}_{y,s}$.

– Assume $p(x-1,y-1)=1$ and $p(x,y-1)<1$.

  * Assume $b_y > 1 - \big(\sum_{s=0}^{y-1} b_s - \sum_{s=1}^{x-1} \overline{b}_{y-1,s}\big)$. Then $v(x,y)=1$ and hence $p(x,y)=p(x-1,y-1)=1$. Thus, we need to show that

$$\sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1} \overline{b}_{y,s} > 1.$$

    If $b_y < \overline{b}_{y-1,x-1}$ this follows from the case assumption; if not, it follows from $p(x-1,y-1)=1$.

  * Assume $b_y \le 1 - \big(\sum_{s=0}^{y-1} b_s - \sum_{s=1}^{x-1} \overline{b}_{y-1,s}\big)$. Then

$$\begin{aligned} p(x,y) &= p(x,y-1) + v(x,y)[1-p(x,y-1)] \\ &= \min\Big\{\sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1} \overline{b}_{y-1,s}, 1\Big\} \end{aligned}$$

    If $b_y < \overline{b}_{y-1,x-1}$ we are done. If not, then we need to show that

$$\sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1} \overline{b}_{y,s} > 1.$$

    But this follows from $p(x-1,y-1)=1$ and $b_y \ge \overline{b}_{y-1,x-1}$.

– Assume $p(x,y-1)=1$. Then $p(x,y)=1$, so we need to show that

$$\sum_{s=0}^{y} b_s - \sum_{s=1}^{x-1} \overline{b}_{y,s} \ge 1.$$

  First notice that $p(x,y-1)=1$ implies

$$\sum_{s=0}^{y-1} b_s - \sum_{s=1}^{x-1} \overline{b}_{y-1,s} \ge 1.$$

  If $b_y > \overline{b}_{y-1,x-1}$, the property follows by adding b_y to the first and second sum of the last equation. Otherwise, it follows by adding b_y to $\sum_{s=0}^{y-1} b_s$ and noting that $\overline{b}_{y,s} = \overline{b}_{y-1,s}$ for $s \le x-1$. □

Proposition 5. *If $C(\cdot)$ is the capture cost (under either simultaneous or sequential voting), $\partial C/\partial n < 0$, $\partial C/\partial m > 0$. Furthermore, if $m < n$ and $\lambda > 1$, $C(\lambda n, \lambda m) < \lambda C(n, m)$.*

Proof. The results for sequential voting follow from a direct inspection of the cost function. For simultaneous voting:

1. $\boxed{\partial C/\partial n < 0.}$ This follows from Proposition 2.

2. $\boxed{\partial C/\partial m > 0.}$ We have

$$\frac{\partial C^{sim}(m)}{\partial m} = -C^{sim}(m)\Big[\log(n-m) - \log(m) + \psi(m) - \psi(n-m+1)\Big].$$

But note

$$\begin{aligned}\psi(m) - \psi(n-m+1) &= \sum_{k=1}^{2m-n-1} \frac{1}{n-m+k} < \sum_{k=1}^{2m-n} \frac{1}{n-m+k} \\ &< \int_0^{2m-n} \frac{1}{n-m+x} dx = \log(m) - \log(n-m),\end{aligned}$$

and thus $\frac{\partial C^{sim}(m)}{\partial m} < 0$.

3. $\boxed{\text{If } m < n \text{ and } \lambda > 1,\ C(\lambda n, \lambda m) < \lambda C(n, m).}$ When λ increases, the number of bribes $n\lambda$ increases linearly, but the value of the bribes decreases. To see that, we fix majority rule $r = \frac{m}{n}$ and consider the bribe for a given λ noted b^*_λ:

$$b^*_\lambda = \binom{\lambda - 1}{r\lambda - 1} r^{r\lambda}(1-r)^{\lambda(1-r)}.$$

Then

$$\begin{aligned}\frac{\partial b^*_\lambda}{\partial \lambda} &= b^*_\lambda\Big[r\log(r) + (1-r)\log(1-r) \\ &\quad + \psi(\lambda) - r\psi(\lambda r) - (1-r)\psi\big(1 + (1-r)\lambda\big)\Big] \\ &= b^*_\lambda\Big[rg(r) + (1-r)g(1-r) - \frac{1}{\lambda}\Big],\end{aligned}$$

where we used the fact that $\psi\big(1+(1-r)\lambda\big) = \psi\big((1-r)\lambda\big) + \frac{1}{(1-r)\lambda}$ and

$$\begin{aligned} g(x) &= \log(x) + \psi(\lambda) - \psi(x\lambda) \\ &= \sum_{k=0}^{(1-x)\lambda-1} \frac{1}{x\lambda+k} - \int_0^{(1-x)\lambda} \frac{1}{x\lambda+k} dk. \end{aligned}$$

For any decreasing function $f(\cdot)$, $f(x) - \int_x^{x+1} f(x)dx < f(x) - f(x+1)$. Setting $f(x) = \frac{1}{x\lambda+k}$, we have

$$g(x) < \sum_{k=0}^{(1-x)\lambda-1} \Big(\frac{1}{x\lambda+k} - \frac{1}{x\lambda+k+1} \Big) = \frac{1}{x\lambda} - \frac{1}{\lambda}.$$

Thus,

$$\frac{\partial b^*_\lambda}{\partial \lambda} < b^*_\lambda \Big[r \left(\frac{1}{r\lambda} - \frac{1}{\lambda} \right) + (1-r) \left(\frac{1}{(1-r)\lambda} - \frac{1}{\lambda} \right) - \frac{1}{\lambda} \Big] = 0. \qquad \square$$

References

Abramowitz, M.; Stegun, I. A. e. (1972). *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (10th ed.).*

Axelrod, R. M. (1970). *Conflict of Interest: A Theory of Divergent Goals with Applications to Politics.* Chicago, IL: Markham Publishing.

Banks, J. S. (2000). Buying supermajorities in finite legislatures. *American Political Science Review 94*(03), 677–681.

Baron, D. P. and D. Diermeier (2001). Elections, Governments, and Parliaments in Proportional Representation Systems. *Quarterly Journal of Economics 116*(3), 933–967.

Baron, D. P. and J. A. Ferejohn (1989). Bargaining in legislatures. *The American Political Science Review*, 1181–1206.

Carrubba, C. J. and C. Volden (2000). Coalitional politics and logrolling in legislative institutions. *American Journal of Political Science 44*(2), 261–277.

Chen, Y. and H. Eraslan (2013). Informational loss in bundled bargaining. *Journal of*

Theoretical Politics 25(3), 338–362.

Chen, Y. and H. Eraslan (2014). Rhetoric in legislative bargaining with asymmetric information. *Theoretical Economics 9*(2), 483–513.

Chen, Y. and J. Zápal (2020). Sequential vote buying. *Available at SSRN 3612957*.

Dahm, M., R. Dur, and A. Glazer (2014). How a firm can induce legislators to adopt a bad policy. *Public Choice 159*(1/2), 63–82.

Dahm, M. and A. Glazer (2015). A carrot and stick approach to agenda-setting. *Journal of Economic Behavior & Organization 116*, 465–480.

Dal Bo, E. (2007). Bribing voters. *American Journal of Political Science 51*(4), 789–803.

Dekel, E., M. O. Jackson, and A. Wolinsky (2008). Vote buying: General elections. *Journal of Political Economy 116*(2), 351–380.

Ekmekci, M. and S. Lauermann (2019). Manipulated electorates and information aggregation. *The Review of Economic Studies*.

Eraslan, H. and K. S. Evdokimov (2019). Legislative and multilateral bargaining. *Annual Review of Economics 11*, 443–472.

Feddersen, T. and W. Pesendorfer (1997). Voting behavior and information aggregation in elections with private information. *Econometrica*, 1029–1058.

Feddersen, T. and W. Pesendorfer (1998). Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political science review 92*(01), 23–35.

Feddersen, T. J. and W. Pesendorfer (1996). The swing voter's curse. *The American economic review*, 408–424.

Felgenhauer, M. and H. Grüner (2008). Committees and special interests. *Journal of Public Economic Theory 10*(2), 219–243.

Genicot, G. and D. Ray (2006). Contracts and externalities: How things fall apart. *Journal of Economic Theory 131*(1), 71–100.

Groseclose, T. and J. M. Snyder (1996). Buying supermajorities. *American Political Science Review 90*(02), 303–315.

Henry, E. (2008). The informational role of supermajorities. *Journal of Public Economics 92*(10), 2225–2239.

Iaryczower, M. and S. Oliveros (2017). Competing for loyalty: the dynamics of rallying support. *American Economic Review 107*(10), 2990–3005.

Koehler, D. H. (1975). Legislative coalition formation: The meaning of minimal winning size with uncertain participation. *American Journal of Political Science*, 27–39.

Lariviere, M. A. (2006). A note on probability distributions with increasing generalized failure rates. *Operations Research 54*(3), 602–604.

Levine, D. K. and T. R. Palfrey (2007). The paradox of voter participation? a laboratory study. *American political science Review*, 143–158.

Louis-Sidois, C. and L. Musolff (2020). Collusion-proof vote buying. *Available at SSRN 3604262*.

Morgan, J. and F. Várdy (2011). On the buyability of voting bodies. *Journal of Theoretical Politics 23*(2), 260–287.

Norman, P. (2002). Legislative bargaining and coalition formation. *Journal of Economic Theory 102*(2), 322–353.

Palfrey, T. R. and H. Rosenthal (1985). Voter participation and strategic uncertainty. *The American Political Science Review*, 62–78.

Rasmusen, E. and J. M. Ramseyer (1994). Cheap bribes and the corruption ban: A coordination game among rational legislators. *Public Choice 78*(3-4), 305–327.

Shaked, M. and J. Shanthikumar (2007). *Stochastic Orders*. Springer Series in Statistics. Springer New York.

Shayo, M. and A. Harel (2012). Non-consequentialist voting. *Journal of Economic Behavior & Organization 81*(1), 299–313.

Shepsle, K. A. and B. R. Weingast (1981). Political preferences for the pork barrel: A generalization. *American Journal of Political Science 25*(1), 96–111.

Spenkuch, J. L., B. P. Montagnes, and D. B. Magleby (2018, July). Backward Induction in the Wild? Evidence from Sequential Voting in the US Senate. *American Economic Review 108*(7), 1971–2013.

Tsai, T.-S. and C. C. Yang (2010). On majoritarian bargaining with incomplete information. *International Economic Review 51*(4), 959–979.

Volden, C. and A. E. Wiseman (2007). Bargaining in legislatures over particularistic and collective goods. *American Political Science Review 101*(1), 7992.

Weingast, B. R. (1979). A rational choice perspective on congressional norms. *American*

Journal of Political Science 23(2), 245–262.

Winter, E. (2004). Incentives and discrimination. *American Economic Review 94*(3), 764–773.

Winter, E. (2006). Optimal incentives for sequential production processes. *The RAND Journal of Economics 37*(2), 376–390.

Zápal, J. (2017). Crafting consensus. *Public Choice 173*(1-2), 169–200.